Lecture 15. Light scattering and absorption by atmospheric particles. Part 3: Scattering and absorption by non-spherical particles: Ray-tracing, T-Matrix, and FDTD methods.

Objectives:

1. Types of nonspherical particles in the atmosphere.
2. Ray-tracing method.
3. Outline of the T-Matrix method.
4. Outline of the FDTD method.

Required Reading:

L02: 5.3, 5.4, 5.5

Advanced/Additional Reading:

Mishchenko, Hovenier, and Travis (Eds.)

Light scattering by nonspherical particles. Academic Press. 2000.

Excellent web site with information on various methods and numerical codes for scattering by nonspherical particles: http://www.t-matrix.de/

1. Types of non-spherical particles in atmosphere.

Recall Lecture 4 about properties of atmospheric aerosol and ice crystals.

Nonspherical aerosols:

Dry salts (e.g., dry sulfates, nitrates, sea-salt)

Dust

Carbonaceous

- In contrast to the spherical particles, the scattering properties of nonspherical particle also depend on shape of the particle and its orientation with respect to the incident light beam.

For non-spherical particles

Recall Lecture 14: In the far-field zone (i.e., at the large distances ***r*** from a particle), the solution of the vector wave equation can be obtained as (eq.[14.12]

$$\begin{bmatrix} E_l^s \\ E_r^s \end{bmatrix} = \frac{\exp(-ikr + ikz)}{ikr} \begin{bmatrix} S_2 & S_3 \\ S_4 & S_1 \end{bmatrix} \begin{bmatrix} E_l^i \\ E_r^i \end{bmatrix}$$

and for the Stokes parameters (see Eq.[14.15]

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{\sigma_s}{4\pi r^2} P \begin{bmatrix} I_o \\ Q_o \\ U_0 \\ V_o \end{bmatrix}$$

where P is the phase matrix

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \qquad [15.1]$$

Orientation of the particles

Aerosol particles have random orientation in space, whereas the ice crystals are often oriented.

Orientation averaged scattering phase function and scattering cross section are

$$P(\Theta) = \frac{1}{2\pi\sigma_s} \int_0^{2\pi} \int_0^{\pi/2} P'(\alpha', \gamma') \sigma_s'(\alpha', \gamma') \sin\alpha' d\alpha' d\gamma' \qquad [15.2]$$

$$\sigma_s = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi/2} \sigma_s'(\alpha', \gamma') \sin\alpha' d\alpha' d\gamma' \qquad [15.3]$$

where α\` and γ\` are the orientation angles of a nonspherical particle with respect to incident light beam.

2. Ray-tracing method.

Ray-tracing method (or geometrical optics approximation, or ray optics approximation) is an approximate method for computing light scattering by particles much larger than a wavelength (i.e., the smallest size parameter is about 80-100).

Basic principles: Ray-tracing method is based on the assumption that the incident EM wave can be represented as a collection of independent parallel rays. Ray tracing is commonly performed using a Monte Carlo approach.

Ray tracing consists of two parts:

1) diffraction theory for the forward scattering peak;
2) ray tracing using Fresnel reflection and transmission formulas.

Advantages: Ray-tracing method can be applied to any shape (spherical or non spherical)

Limitations: Ray-tracing method is an approximate by definition;

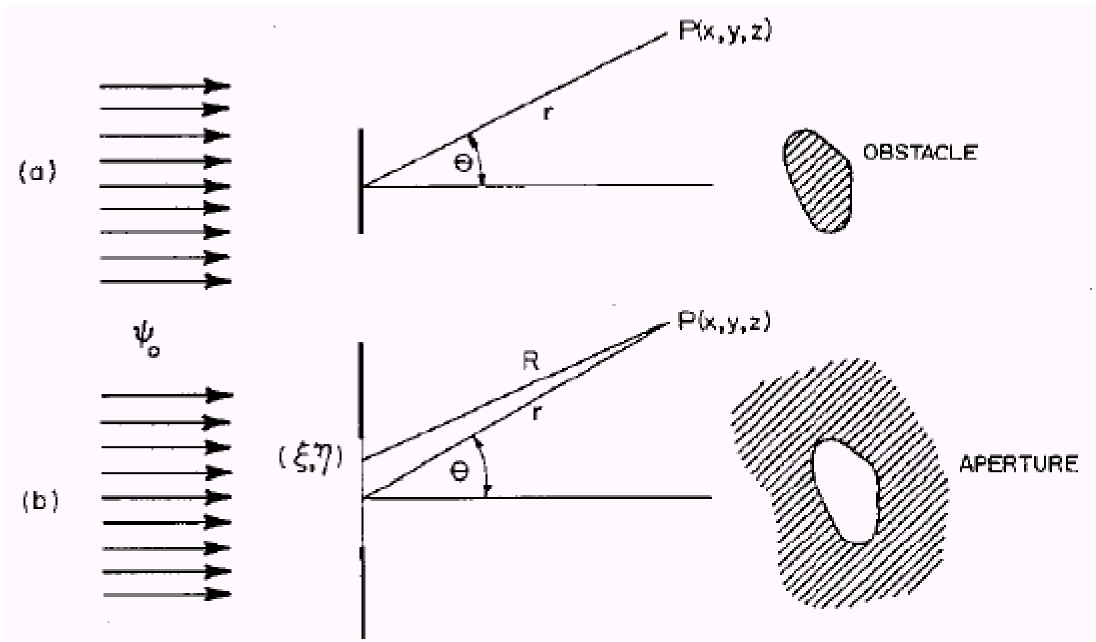
Limited range of size parameters;

Absorbing particles require special treatment.

➢ ***Diffraction***

In geometric optics, light may be treated as rays, except for Fraunhofer diffraction around a particle.

In geometric optics limit ($x \gg 1$) light may be treated as rays, except for Fraunhofer diffraction around a particle.



Babinet's principle- diffraction pattern is the same from an aperture as for opaque particle of same size.

Integrate the far field contribution of incident wave over the aperture

$$E = -\frac{iE_0}{R\lambda}\int_A \exp(-ikR)\,dA \qquad [15.4]$$

Huygens principle – each point is a source of circular wave fronts.

For sphere (circular aperture), the diffraction pattern is

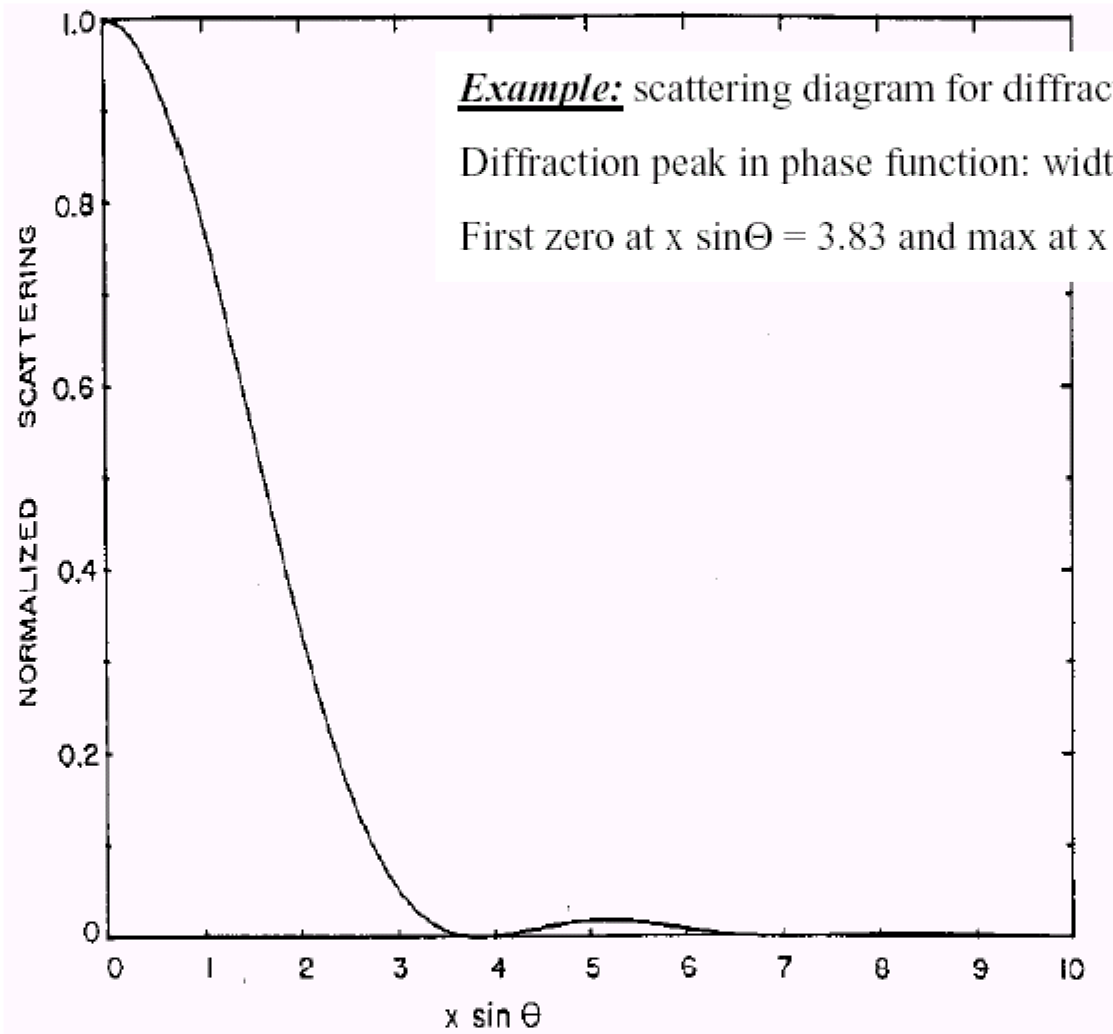
$$I(\Theta) = -\frac{I_o}{R^2k^2}\frac{x^4}{4}\left[\frac{2J_1(x\sin\Theta)}{x\sin\Theta}\right]^2 \qquad [15.5]$$

where $k = 2\pi/\lambda$, and $\boldsymbol{J}_1$ is Bessel function.

Example: scattering diagram for diffraction by a circular disk

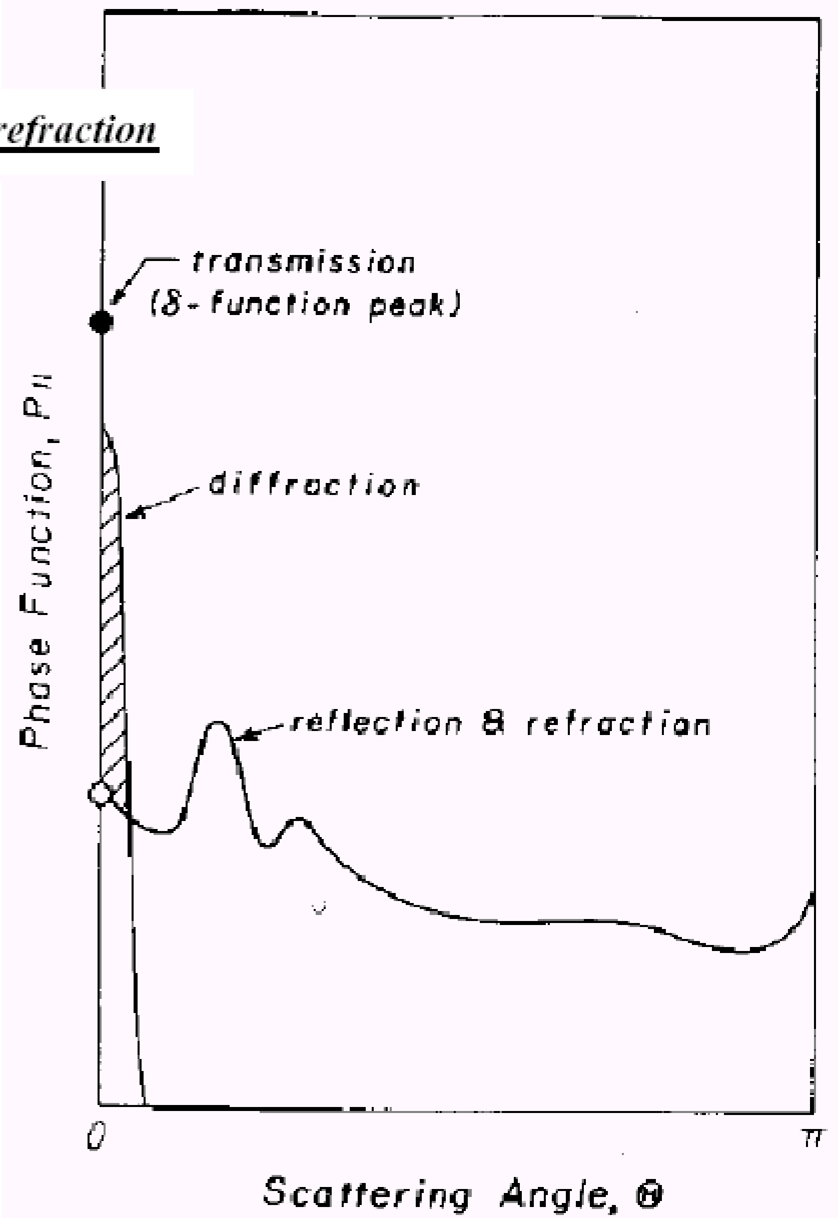
Diffraction peak in phase function: width $\Theta \sim 1/x$, height $P(0) \sim x^2$

First zero at $x \sin\Theta = 3.83$ and max at $x \sin\Theta = 5.14$

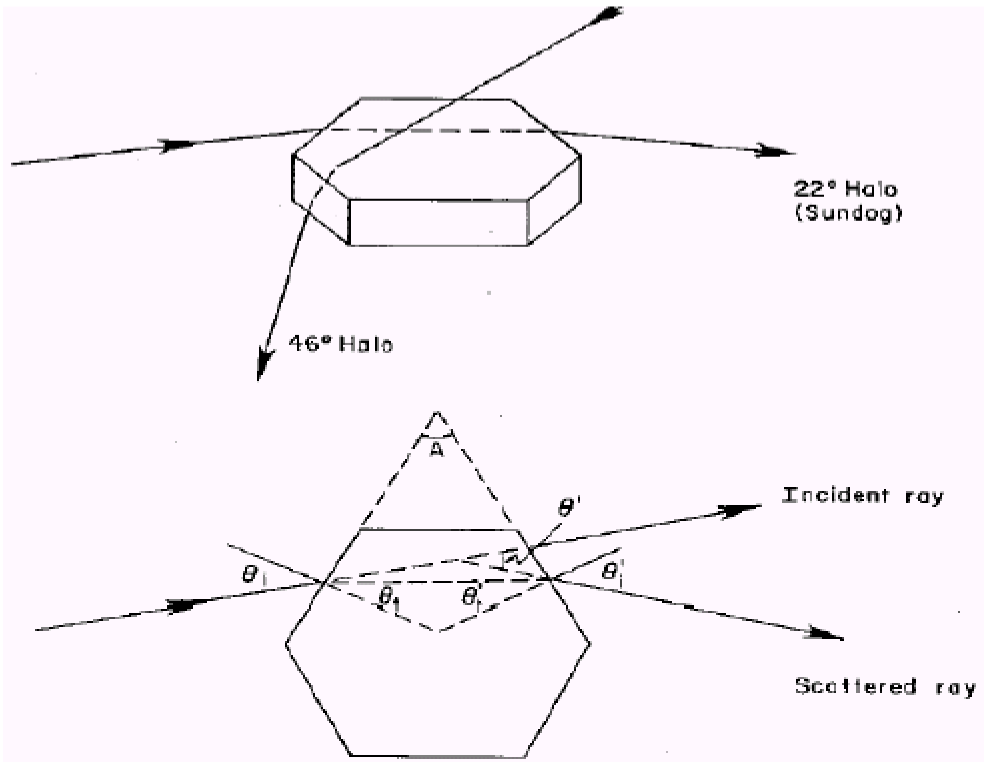


➢ ***Diffraction + reflection and refraction***

Fig. 15.1 A schematic representation of the components of the phase function P11 from randomly oriented hexagonal ice crystals (from Liou, 1992).



➢ ***Fresnel reflection and transmission***



Snell's law gives refraction angle θ_t:

$$\sin \theta_i = m \sin \theta_t \qquad [15.6]$$

Fresnel formula for polarized reflection amplitude coefficients:

$$r_r = \frac{\cos\theta_i - \sqrt{m^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{m^2 - \sin^2\theta_i}} \qquad [15.7]$$

$$r_l = \frac{\sqrt{m^2 - \sin^2\theta_i} - m^2\cos\theta_i}{\sqrt{m^2 - \sin^2\theta_i} + m^2\cos\theta_i} \qquad [15.8]$$

Reflectivity and transmission coefficient for intensity:

$$R_r = \left|r_r\right|^2, \quad R_l = \left|r_l\right|^2, \qquad T_r = 1 - R_r, \quad T_l = 1 - R_l \qquad [15.9]$$

NOTE: For $\theta = 0^0$, reflection is $R = \left|\frac{m-1}{m+1}\right|^2$ => reflectivity increases with refractive

index

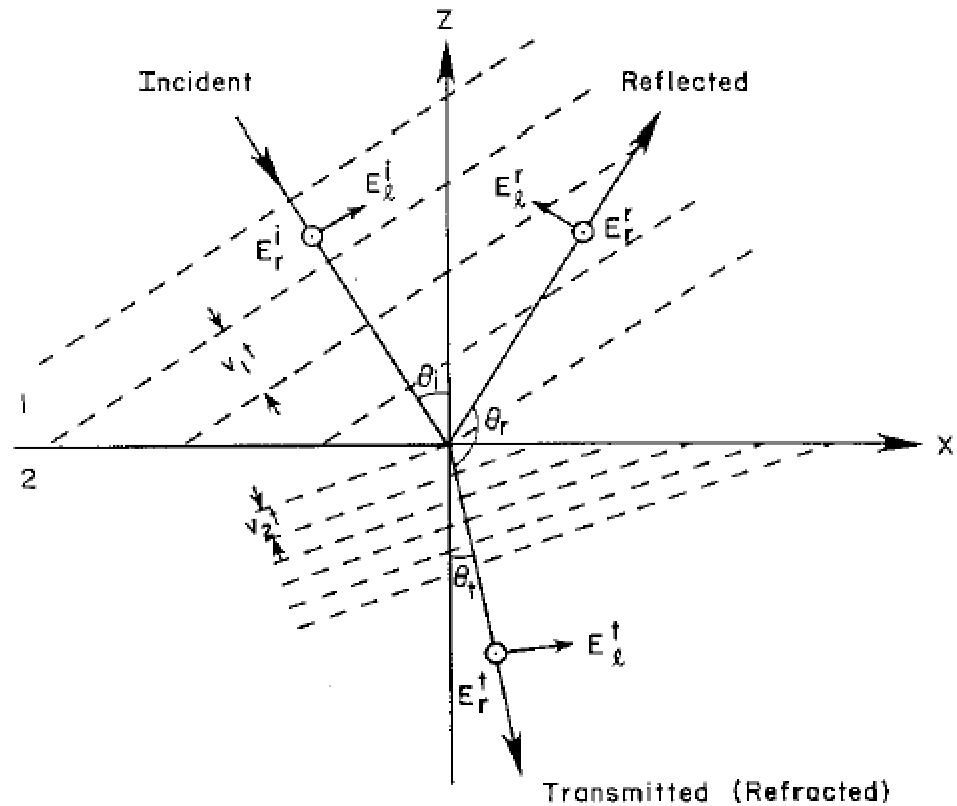
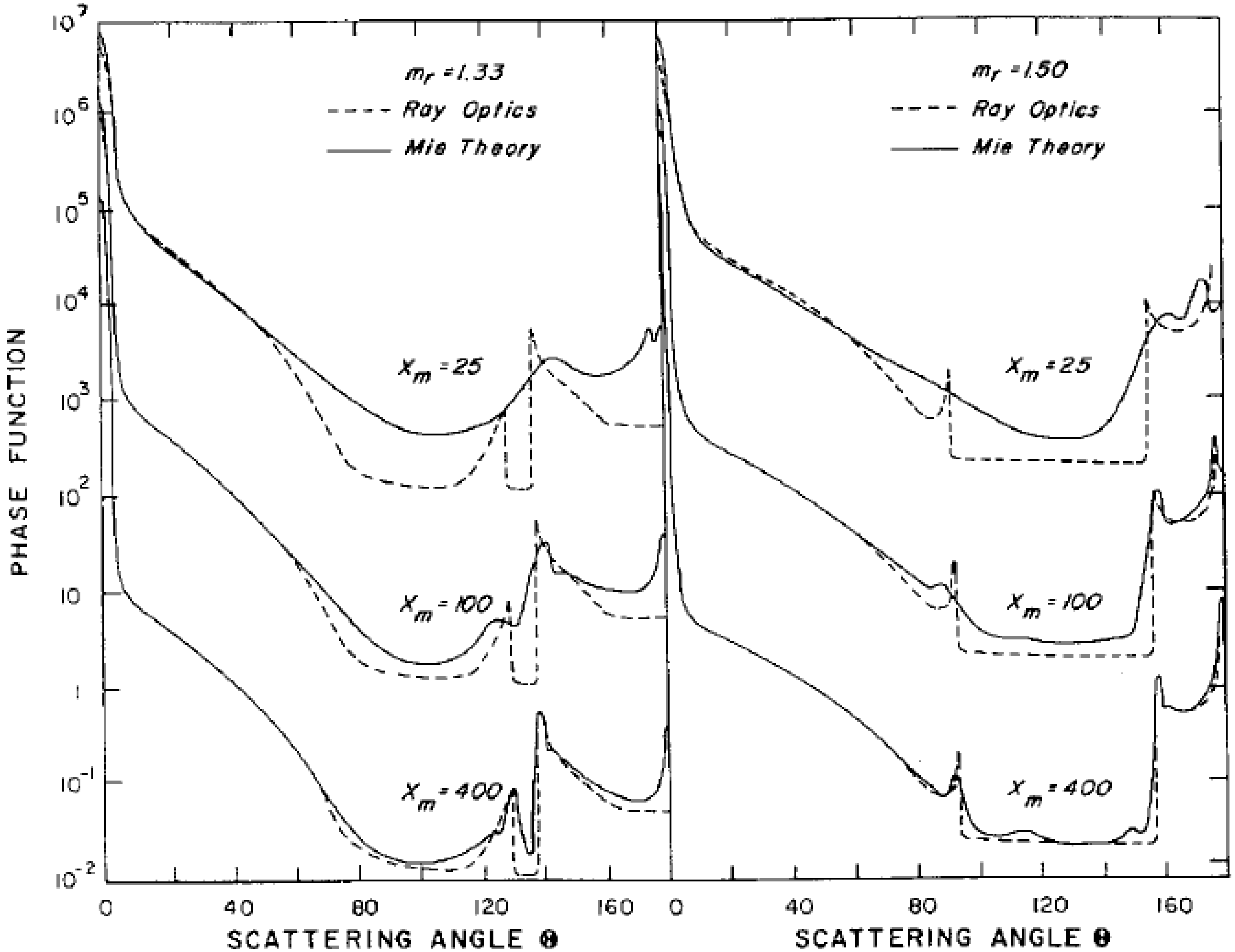


Illustration of the reflection and refraction of a plane wave. The choice of the positive directions for the parallel components (ℓ) of the electric vectors is indicated in the diagram. The perpendicular components are at right angles into the plane of reference. [Liou, 1994; Fig. 5.8]

Ray Tracing Results.

Ray tracing compares well with Mie theory for large size parameters:

- For water drops in the visible ($m = 1.33$) the primary rainbow is at $\Theta = 137°$ from one internal reflection.
- Rainbow angle depends on index of refraction through Snell's Law.
- Ray tracing cannot explain glory (at $\Theta = 180°$) for cloud droplets.



Comparison of ray tracing and Mie phase functions for spheres. Two refractive indices are shown along with three size distributions; the vertical scale is shifted by 10^2 and 10^4 for the $X_m = 100$ and $X_m = 25$ curves. [Liou, 1980; Fig. 5.11]

Visible light ray tracing in randomly oriented hexagonal ice crystals explains 22° and 46° halos.

Modeled and measured nonspherical phase functions for ice crystals show that Mie theory gives too little side scattering.

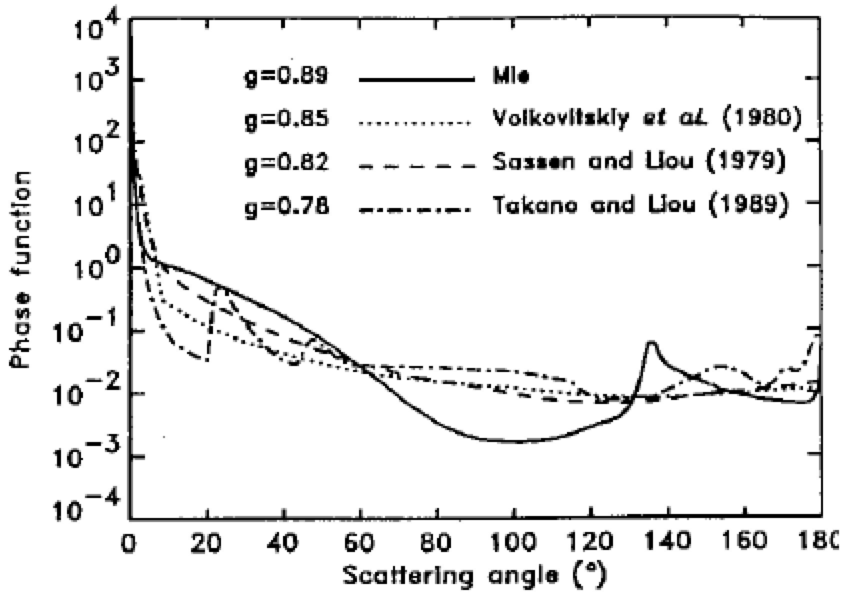


FIG. 7. Phase functions used in this study, including the corresponding values of the asymmetry parameter g.

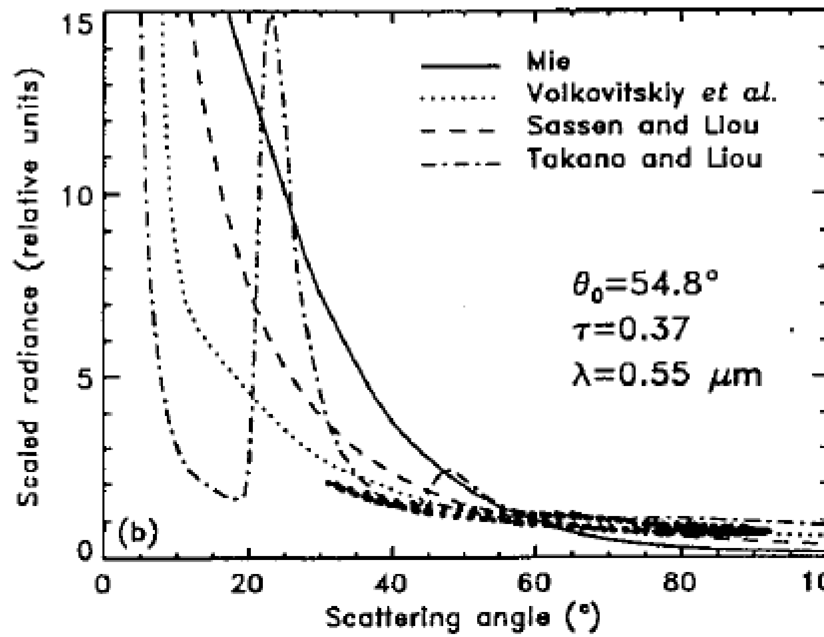


FIG. 8. (a) Comparisons between measured radiances (●) and Monte Carlo simulations for A189 orbit 1 (30°) at a wavelength of 0.55 μm. (b) As in (a) but for A189 orbit 2 (45°).

Mie and modeled or lab measured phase functions for cirrus cloud ice crystals. (from Francis, 1995: Some Aircraft Observations of the Scattering Properties of Ice Crystals. J. Atmos. Sci., 52, 1142.)

3. Outline of the T-Matrix method.

T-Matrix method, TMM, enables calculation of optical properties of particles with rotationally symmetric shape (such as ellipsoids, circular cylinders, Chebyshev shapes, etc.)

NOTE: FORTRAN code of TMM is openly available at http://www.giss.nasa.gov/~crmim

Basic principles: TMM is based on expanding the incident EM and scattered fields in vector spherical wave functions. The T matrix transforms the expansion coefficients of the incident field into those of scattered field and, if known, can be used to compute any scattering characteristic of a nonspherical particle. The elements of the T matrix are independent of the incident and scattering fields and depend only on the shape, size parameter, and refractive index of the scattering particle and on its orientation with respect to the reference frame.

Advantages: TTM is highly accurate and computationally fast

Limitations: Limited types of particle shapes;

Limited range of size parameters ($x < 30$).